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TECHNICAL NOTE NO. 27-81

AN INVESTIGATION OF THE LINK PERFORMANCE OF CRISIS CALLS

DECEMBER 1981

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In this technical note we develop a mathematical model for the link performance of crisis calls. In such a system a finite number of crisis calls attempt to use the link, which has been receiving ordinary calls. The performance of the link with respect to the crisis calls is studied. A numerical analysis is conducted to determine the behavior of the crisis calls. Some simple upper and lower bounds on the blocking and loss probabilities for crisis calls are given.

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AN INVESTIGATION OF THE LINK PERFORMANCE

OF CRISIS CALLS

DECEMBER 1981

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Approved for Publication

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FOREWORD

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EXECUTIVE SUMMARY

Periodically DCEC is tasked by DCA Headquarters to conduct what has been commonly known as the "Flash Non-Blocking Study". The purpose of this study is to resize the AUTOVON network to be sure the crisis calls receive non-blocking service. The main tool that has been used to conduct this study is a simulation (event by event) model. This model is very computer expensive. Typically, 50 hours of CPU time are required to conduct the study.

DCEC has a mathematical network sizing model that both DCEC and DCA headquarters use to resize the AUTOVON for its day to day operation. This model is extremely fast and requires only a few minutes of CPU to do a particular sizing run. Because of characteristics of the crisis traffic this model cannot be used in its current form for the Flash Non-Blocking studies. This technical note is the first step in an attempt to modify the mathematical model so it can be used on these studies.

The mathematical model requires the ability to simply predict the link performance of the class of traffic under consideration. In this technical note we develop a sophisticated analytic link performance model and then use it in a numerical analysis of the link behavior of crisis calls. This investigation leads to the development of simply computable bounds which could be used in our mathematical network sizing model.

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I. INTRODUCTION

Periodically, the Defense Communications Engineering Center (DCEC) conducts a study for the Defense Communications Agency (DCA), the "Flash Non-Blocking Study" [1]. In this study a crisis scenario for the worldwide AUTOVON system is analyzed and the network resized to ensure the crisis calls are not blocked in the network. This scenario postulates a certain number and distribution of crisis calls that would have to be placed for the particular situation being analyzed. These calls must receive non-blocking performance from the AUTOVON network. The purpose of the study is to resize the links to be certain that the required grade of service is received.

For normal resizing efforts of AUTOVON, DCEC uses its Network Design and Analysis computer algorithm [2]. The current form of the algorithm can not be used for the Flash Non-Blocking Studies because the arrival rate and holding time characteristics of the crisis calls do not conform to the algorithms assumptions for a normal AUTOVON call. Thus DCEC has been forced to use an event by event simulation model to conduct the study.

After a typical simulation run, the engineer checks to see if the crisis calls are receiving non-blocking service. If not, channels would be added and deleted in the network based on engineering judgement and then another simulation run is submitted. This trial and error process is continued until the crisis calls receive the required non-blocking service. There is no guarantee that this solution is the most economical one possible (i.e., optimum). For the normal resizing of AUTOVON via the DCEC Network Design and

Analysis algorithm this process is done entirely by the computer and a near optimum solution is provided.

For each simulation run several replications have to be made because there are only a finite number of crisis calls trying to use the network and the multiple replications are required to attain statistical confidence in the grade of service estimates from the simulation run. Thus, a significant amount of computer CPU time is required; a typical Flash Non-Blocking study consumes about 50 hours of CPU time on the DCEC ITEL-AS5 computer.

This technical note is an initial effort to reduce this computer run time and provide a better tool to conduct the Flash Non-Blocking Type Studies. The Network Design and Analysis algorithm has a mathematical network performance model [3] that iteratively converges to the performance of the network under consideration. The computer run time of this performance model is significantly shorter (order of magnitude)than a comparable simulation run and hence would be desirable for use on the Flash Non-Blocking studies. It converges to the network grade of service by considering the blocking on each link of the network. Since no mathematical link performance model was available which characterized the crisis calls, the network performance model could not be used to address Flash Non-Blocking issues in the past.

In this technical report we present a mathematical link performance model for this system. Section II of this report describes the mathematical model, as well as some special cases. Several numerical examples are given in section III. Finally, a few concluding remarks are given in section IV.

II. A MATHEMATICAL MODEL

1. ASSUMPTIONS AND GENERAL DESCRIPTION

In this section we present a mathematical model for the performance of crisis calls on a link of a communications network where ordinary calls are usually using the channels. We assume that there are C channels, the arrival process of the ordinary calls is Poisson with parameter λ, and their holding time is exponentially distributed with mean $1/\mu$. There are a finite number, say M, of crisis calls that are going to attempt to use these channels. The interarrival time between the m^{th} and $(m+1)^{st}$ crisis call has a distribution function, $G(x)$, with mean $1/\alpha$.

No queueing of either class of calls is allowed and if a crisis call seizes a channel, it will hold the channel for at least as long as it takes to have all the M crisis calls attempt to use the channels. Thus, if the m^{th} crisis call occupies a channel, it will still be holding the channel when the M^{th} crisis call attempts to find a full channel. We investigate the behavior of the system with respect to the crisis calls.

Loss systems similar to this one have been considered in the past. The main paper of interest is that of Kuczura [4]. In that paper the author considers a loss system with two types of input processes, one Poisson and the other a general renewal process. He shows that the blocking probability each stream sees is different. Our system differs from his in the following ways.

First, one of our streams has only a finite number of calls that attempt to use the channels; in Kuczura's paper there are basically an infinite number of calls. The second difference is in the nature of how long the crisis calls hold the channels once they seize a channel. In Kuczura's paper it was assumed that both classes of calls had the same exponentially distributed holding time distribution. In our system the ordinary calls have the standard exponentially distributed holding time, but the crisis calls hold the channels for a period of time that is at least as long as it takes all crisis calls to arrive. The final way our system differs from Kuczura's is that we allow the crisis calls to preempt an ordinary call. We let β be the probability an arriving blocked crisis call will preempt a ordinary call using the channels. For a network interpretation of this quantity, see reference [5].

Another paper, by Fischer [6], considers a system similiar to the one examined here. In that paper the author studies a loss system with two classes of customers with priorities. The effects of different mean holding times for each class of traffic is studied. That system differs from ours in that we only have a finite number of calls that are allowed to preempt and they only try once during the period of interest. Furthermore, the nature of the holding time for the crisis calls is different in our case.

We have M crisis calls attempting to use the C channels. These channels are currently being used by ordinary calls with a load, $\rho=\lambda/\mu$. We are interested in studying the behavior of the system with respect to the crisis calls during the period of time the M crisis calls attempt to use the channels. In order to do so we need some transient results for a standard

Markovian loss system with C channels, call arrival rate λ and service rate μ. That is we need the transient behavior of the system with only ordinary calls.

Let Q(t) be the number of ordinary calls in the system at time t and $r_{i,j}(t)=\Pr\{Q(t)=j|Q(0)=i\}$; then from Riordan [7], when there are no crisis calls arriving in [0,t], we have for i=0, 1, ..., C, and j=0, 1, ..., C

$$r_{i,j}(t) = b_j + \sum_{k=1}^{C} b_k(i,j)e^{\theta_k \mu t} \tag{1}$$

where

$$b_j = \frac{\frac{\rho^j}{j!}}{\sum_{r=0}^{C}\frac{\rho^r}{r!}}, \tag{2}$$

$$b_k(i,j) = \frac{C!\rho^{C-i}F_i(\theta_k)F_j(\theta_k)}{j!\theta_k F_C(\theta_k)F_C'(\theta_k+1)} \tag{3}$$

with

$$F_0(z) = 1$$

$$F_i(z) = \sum_{k=0}^{i} \binom{i}{k}\rho^{i-k}z(z+1)\ldots(z+k-1). \tag{4}$$

The quantity $F_C'(\theta_k+1)$ in equation (3) is the derivative of $F_C(z)$ evaluated at θ_k+1. The factors θ_k, k=1, 2, ..., C are the roots of the equation

$$F_{\ell}(z+1)\,\mu. \tag{5}$$

Let $p_{i,j}^{(m)}$ be the probability the number of ordinary calls in the system is i and the number of crisis calls is j just before the m^{th} crisis call arrives. We will evaluate $p_{i,j}^{(m)}$ from $p_{i,j}^{(m-1)}$ but to do so we need to determine the number of ordinary calls in the system when the m^{th} crisis call arrives, given the number in the system at the $(m-1)^{st}$ arrival. Since the interarrival distribution time between two successive crisis calls is $G(x)$, let us define its Laplace transform by $\phi(s)$ as

$$\phi(s) = \int_0^\infty e^{-sx}\,dG(x). \tag{6}$$

Now consider what happens when the $(m-1)^{st}$ crisis call arrives; if the number of crisis calls already there is j-1 and it joins them, (i.e., there is a free channel or it preempts an ordinary call), then there are j crisis calls in the system just after it arrived. Until the m^{th} crisis call arrives there are C-j channels for use by the ordinary calls. Thus between any two successive crisis call arrivals the number of channels available to the ordinary calls, denoted by n, could be n=0, 1, ..., C-j. The probability $p_{i,j}^{(m)}$ will determine how many channels are available after the $(m-1)^{st}$ arrival.

We can use $r_{i,j}(t)$ and $G(x)$ to mathematically describe the behavior of the ordinary calls during this time. Let us suppose the $(m-1)^{st}$ crisis call arrives at t_{m-1} and the (m^{th}) at t_m and during this time there are n channels available to the ordinary calls. Let $R_{i,j}[n]$ be the probability $Q(t_m)=j$ given $Q(t_{m-1})=i$;

then using (1)

$$R_{i,j}[n] = \int_0^\infty r_{i,j}(t)dG(t)$$

$$= b_j + \sum_{k=1}^{n} b_k(i,j)\phi(-\theta_k\mu). \qquad (7)$$

We assume that before the crisis calls start to arrive the system has reached steady state with respect to the ordinary calls. Furthermore, we assume the initial crisis call sees the same distribution of the number of ordinary calls in the system as an arriving ordinary call, so

$$P_{i,j}^{(1)} = \begin{cases} b_i & j=0 \\ 0 & j\geq 1 \end{cases} \qquad (8)$$

where b_i is given by equation (2).

A general recursion for $P_{i,j}^{(m)}$ when $m\geq 2$ can now be given as

$$P_{i,j}^{(m)} = \begin{cases} P_{C,0}^{(m-1)} (1-\beta) R_{C,i}[C] & j=0 \\ \sum_{k=0}^{C-j} P_{k,j-1}^{(m-1)} R_{k,i}[C-j] + \beta P_{C+1-j,j-1}^{(m-1)} R_{C-j,i}[C-j] \\ \qquad + (1-\beta) P_{C-j,j}^{(m-1)} R_{C-j,i}[C-j] & 1 \le j \le C-i \\ P_{0,C-1}^{(m-1)} + P_{0,C}^{(m-1)} + \beta P_{1,C-1}^{(m-1)} & j=C \end{cases} \tag{9}$$

where $0 \le i+j \le C$ and $P_{i,j}^{(m)}=0$ if $j \ge m$. Thus from equations (8) and (9) we can recursively evaluate $P_{i,j}^{(m)}$.

There are two measures of performance with respect to the crisis calls: the probability the m^{th} crisis call is blocked, $PB^{(m)}$, and the probability the m^{th} crisis call is lost. The probability $PB^{(m)}$ represents the probability the m^{th} crisis call sees all the channels busy. Whereas, $PL^{(m)}$ represents the disposition of the call upon arrival to the channels. We have

$$PB^{(m)} = \sum_{i=0}^{C} P_{i,C-i} \tag{10}$$

and

$$PL^{(m)} = (1-\beta) PB^{(m)} + \beta P_{0,C}. \tag{11}$$

We note when $\beta=0$ $PB^{(m)}=PL^{(m)}$ and define the average blocking and loss probabilities to be $PB=\sum_{m=1}^{M} PB^{(m)}/M$ and $PL=\sum_{m=1}^{M} PL^{(m)}/M$.

In general, a closed form expression for $P_{i,j}^{(m)}$ that satisfies equation (9) is hard to obtain. But, using that equation and equation (8), the values of $P_{i,j}^{(m)}$ can be iteratively computed from $P_{i,j}^{(m-1)}$ using a computer. There are however some special uses that allow us to obtain more insight into the behavior of the system.

2. THE SINGLE CHANNEL CASE (C=1)

In this case there is only one channel and so

$$P_{i,j}^{(1)} = \begin{cases} \dfrac{\frac{\rho^i}{i!}}{1+\rho} & j=0 \\ 0 & j=1 \end{cases} \tag{12}$$

From equation (9), for $m \geq 2$ we have

$$\begin{aligned} P_{00}^{(m)} &= \frac{\rho}{1+\rho} R_{11}[1]^{m-2} R_{10}[1] (1-\beta)^{m-1} \\ P_{10}^{(m)} &= \frac{\rho}{1+\rho} R_{11}[1]^{m-1} (1-\beta)^{m-1} \\ P_{01}^{(m)} &= 1-\frac{\rho}{1+\rho} R_{11}[1]^{m-2} (1-\beta)^{m-1} . \end{aligned} \tag{13}$$

Since $R_{10}[1] = 1-R_{11}[1]$, $P_{i,j}^{(m)}$ only depends on

$$R_{11}[1] = \frac{\rho}{1+\rho} + \frac{\rho(1-\mu\theta_1)}{1+\rho}; \qquad (14)$$

but for $C=1$ we have $\theta_1=-1-\rho$, see [7], and so

$$R_{11}[1] = \frac{\rho}{1+\rho} + \frac{\rho([illegible])}{1+\rho}.$$

For the m^{th} customer the two measures of performance are his blocking probability, $PB^{(m)}$ and loss probability, $PL^{(m)}$. For $m \geq 2$, we have

$$\begin{aligned} PB^{(m)} &= P_{10}^{(m)} + P_{01}^{(m)} \\ &= 1-P_{00}^{(m)} \qquad (16) \\ &= 1-\frac{\rho}{1+\rho} R_{11}[1]^{m-2} R_{10}[1](1-\beta)^{m-1} \end{aligned}$$

and a loss probability, $PL^{(m)}$

$$\begin{aligned} PL^{(m)} &= (1-\beta)\, PB^{(m)} + \beta P_{01}^{(m)} \\ &= PB^{(m)} - \beta[P_{10}^{(m)} + P_{01}^{(m)}] + \beta P_{01}^{(m)} \\ &= PB^{(m)} - \beta P_{10}^{(m)} \qquad (17) \\ &= (1-\beta)P_{10}^{(m)} + P_{01}^{(m)} \\ &= 1-\frac{\rho}{1+\rho} R_{11}[1]^{m-2}(1-\beta)^{m-1}(R_{10}[1]+\beta R_{11}[1]). \end{aligned}$$

For the case of m=1 the desired results are

$$PB^{(1)} = \frac{\rho}{1+\rho}$$

$$P_L(1) = \frac{(1-\beta)\rho}{1+\rho}. \tag{18}$$

Using equation (17) we have

$$PB = PL + \frac{\beta\rho\{1-(R_{11}[1](1-\beta))^{M-1}\}}{M(1+\rho)\ \{1-R_{11}[1](1-\beta)\}}, \tag{19}$$

where

$$PL = \frac{1}{M}\{M-1 \frac{+\rho(1-\beta)^M R_{11}[1]^{M-1}}{1+\rho}\}. \tag{20}$$

These results can be even further simplified for the cases where $\beta=0$ and $\beta=1$. When $\beta=0$ the crisis calls does not return to preempt when blocked and

$$PL = \frac{1}{M}\left\{M-1+\rho\frac{R_{11}[1]^{M-1}}{1+\rho}\right\} \tag{21}$$

$$= PB.$$

When $\beta=1$ the crisis calls always return to preempt when blocked and we have

$$PL = \frac{M-1}{M} \tag{22}$$

and

$$PB = \frac{1}{M}\left\{M-1+\frac{\rho}{1+\rho}\right\}.$$

Equation (22) agrees with one's intuition; all but the first crisis call is lost and the first is blocked with probability $\rho/(1+\rho)$.

Several further results can be derived for this system by considering the case where $\beta=0$. For that situation we have

$$PB^{(m)} = PL^{(m)}$$

$$= \begin{cases} \dfrac{\rho}{1+\rho} & m=1 \\ 1 - \dfrac{\rho}{1+\rho}(R_{11}[1])^{m-2} R_{10}[1] & m \geq 2 \end{cases} \tag{23}$$

and

$$PB = PL$$
$$= \frac{1}{M}\left(M-1 + \frac{\rho R_{11}[1]^{M-1}}{1+\rho}\right). \tag{24}$$

It is straightforward to show, using equation (23), that for m=1, 2, ..., M

$$PB^{(m)} \leq PB^{(m+1)} \tag{25}$$

and $\lim_{m\to\infty} PB^{(m)} = 1$. Thus the $(m+1)^{st}$ crisis call sees a higher blocking than the m^{th} crises call. Of course, the unequality makes sense, since the crisis calls that contribute to the m^{th} call blocking will also contribute to the $(m+1)^{st}$ crisis call blocking. If the m^{th} call gets in then it also contributes to the blocking of the $(m+1)^{st}$ call, and since the interarrival distribution between two successive crisis calls is the same, the probability the $(m+1)^{st}$ crisis call gets blocked is greater than the probability the m^{th} call is blocked.

The other results follow from equation (24) and is analogous to the result established in Kuczura [4]. In that paper he considers a loss system with two classes of arrivals, one whose arrival process was described by a Poisson process and the other by a general renewal process. He shows, using Jensen's inequality, that the blocking the general arrivals sees is minimized when their interarrival distribution is deterministic. From equation (24) we see that PB linearly depends on $R_{11}[1]$ where

$$R_{11}[1] = \frac{\rho}{1+\rho} + \frac{\phi(\mu+\lambda)}{1+\rho}.$$

Remembering that $\phi(s)$ is the Laplace transform of the interarrival distribution of successive crisis calls, then from Jensen's inequality, $R_{11}[1]$ is the smallest when that distribution is deterministic. The distribution that minimizes $R_{11}[1]$ also minimizes PB.

3. MEAN TIME BETWEEN CRISIS CALL ARRIVALS IS ZERO ($\alpha=\infty$)

For this case and the next one we assume the interarrival distribution of successive crisis calls is exponential. Here, the crisis calls all show up at the same time and since they see the same distribution as the ordinary calls, the probability the m^{th} crisis call is blocked is the probability there are $(C-m+1)$ or more ordinary calls present. So for $m \leq C+1$ we have (Q is the steady state number of ordinary calls in system)

$$PB^{(m)} = \sum_{r=C+1-m}^{C} P_r\{Q=r\}$$

$$= \frac{\sum_{R=C+1-m}^{C} \rho^r/r!}{\sum_{R=0}^{C} \rho^k/k!} \tag{26}$$

$$= \frac{\sum_{r=0}^{m-1} \rho^{C-r}/(C-r)!}{\sum_{k=0}^{C} \rho^k/k!}$$

and $PB^{(m)}=1$ for $m \geq C+1$.

The expression for $PB^{(m)}$ given by equation (26) does not depend on the capability of evaluating $P^{(m)}_{i,C-i}$, for i=0, 1, ...,C. Unfortunately, developing an expression for $PL^{(m)}$ necessitates a knowledge of what $P^{(m)}_{0,C}$ is. In general

$$PL^{(m)} = (1-\beta)PB^{(m)} + P^{(m)}_{0,C},$$

but $P^{(m)}_{0,C}$ equals the probability there are C crisis calls in the system just after the $(m-1)^{st}$ crisis call arrives. Let $U_r^{(m)}$ be the probability there are r crisis calls in the system just after the m^{th} crisis call arrives and decides whether or not to preempt. Define $U_0^{(0)}=1$ and $U_r^{(m)}=0$ for r>m; then

$$U_0^{(m)} = U_0^{(m-1)} PL^{(m)}$$

$$U_r^{m} = U_{r-1}^{(m-1)} (1-PL^{(m)}) + U_r^{(m-1)} PL^{(m)} \qquad 1 \le r \le m-1 \qquad (27)$$

$$U_m^{(m)} = U_{m-1}^{(m-1)} (1-PL^{(m)})$$

iteratively defines $U_r^{(m)}$ in terms of $U_r^{(m-1)}$ and $PL^{(m)}$.

The loss probability for the m^{th} crisis call can be given by

$$PL^{(m)} = \begin{cases} E_B(\rho,C)(1-\beta) & m=1 \\ (1-\beta)PB^{(m)}+\beta U_C^{(m-1)} & m\ge 2 \end{cases} \qquad (28)$$

where $E_B(\rho,C)$ is Erlang's Loss Formula. So we can use equation (28) to evaluate $PL^{(1)}$, then equation (27) to recursively determine $U_r^{(1)}$, r=0,1. These probabilities can be used via equation (28) to determine $PL^{(2)}$ and so on.

4. MEAN TIME BETWEEN CRISIS CALL ARRIVALS IS INFINITE (α=0)

For this case the average time between successive crisis call arrivals is infinite. This implies that, after the last crisis call arrives and there are r crisis calls in the system, the probability the next crisis call sees i ordinary calls present is given by

$$R_{k,i}[C-r]=\frac{\frac{\rho^{i}}{i!}}{\sum_{m=0}^{C-r}\frac{\rho}{r!}} \qquad i=0,1,\ldots,C-r. \tag{29}$$

independent of k.

Using the probabilities defined in equations (27) and (28) we have

$$\begin{aligned} PB^{(m)} &= \sum_{r=0}^{m-1} U_r^{(m-1)} E_B(\rho, C-r) \\ PL^{(m)} &= (1-\beta)PB^{(m)} + \beta U_C^{(m-1)} \quad . \end{aligned} \tag{30}$$

Although difficult to show mathematically, it is intuitively true that the values of $PB^{(m)}$ and $PL^{(m)}$ given by equations (26) and (28) serve as upper bounds on these probabilities for an arbitrary value of α. Furthermore, the values given by equations (30) are lower bounds on $PB^{(m)}$ and $PL^{(m)}$ for any α. This statement is demonstrated in the numerical examples we present in the next section.

III. SOME NUMERICAL EXAMPLES

In all the numerical examples that we considered, we have assumed the interarrival distribution between successive crisis calls is exponentially distributed with mean α^{-1}. Thus $\phi(-\theta_k u)$ in equation (7) is given by

$$\phi(-\theta_k\mu)=\frac{\alpha}{\alpha-\theta_k\mu} . \tag{31}$$

In this section we examine PB and PL as a function of α and β . We also present some numerical results for $PB^{(m)}$ and $PL^{(m)}$ as a function of β.

For a given value of λ, μ, α, C and M several significant numerical problems have to be overcome before $P_{ij}^{(m)}$ can be recursively evaluated using equation (9). The first problem is the determination of the roots of n roots $\theta_1, \theta_2, \cdots, \theta_n$ of the equation

$$F_n(z+1)=0,$$

where n=0,1,...,C. We used the Laguerre's iteration method [8] as suggested by Kuczura [4] to find these roots. We also followed some of the helpful numerical hints Kuczura discovered when he was forced to find these roots. Even for moderately small values of C, say C=20, the behavior of the function $F_C(z+1)$ is extremely erratic and care must be taken in any numerical procedure attempting to find the roots. Finally, it should be pointed out that the function $F_C(z)$ and hence $F_C'(z)$ can be recursively evaluated using the relations given in [4] or [7]. Finding the roots poses the biggest numerical problem one has to face. The remaining computational steps are straightforward in determining $P_{ij}^{(n)}$.

Table I presents the sensitivity of $PB^{(m)}$ and $PL^{(m)}$ to the probability of preemption. For that table, C=15, M=20, λ=15 erlangs and α=10. When β=0 we had $PB^{(m)}=PL^{(m)}$, but as β approached 1, $PL^{(m)}$ was approaching zero for $m \leq C=15$ and 1 for $m > C=15$. It is interesting to see what happens to $PB^{(m)}$ as $\beta \to 1$. For m small the value of $PB^{(m)}$ remains relatively constant as $\beta \to 1$; but as m approaches M this is not true. Table I also demonstrates a result we gave for the case C=1; that is, $PB^{(m)} \leq PB^{(m+1)}$. For this table we see that $PB^{(m)}$ and $PL^{(m)}$ are both monotonically increasing in m.

Figure 1 gives a family of curves for the average blocking and loss probability as a function of β for ordinary loads of 5, 15 and 25 erlangs. The solid lines are the blocking probability and for each of the three cases it appears that PB is a linear function of β. The dashed lines give the average loss probability. At β=0 we have PB=PL and when β=1 all three curves for the average loss probability converge to the same value (.33). As expected, PB is monotonically increasing in β whereas PL is monotonically decreasing. With regard to PL, it is interesting to point out that its lower bound is (M-C)/M for $M \geq C$ and the upper bound is PB at β=0.

TABLE I. SENSITIVITIES OF INDIVIDUAL BLOCKING AND LOSS PROBABILITIES

TO THE PROBABILITY OF PREEMPTION

(C=15, M=20 =15 and =10)

β	0		.2		.4		.6		.8		1.0	
m	$\overline{PB}(m)$	$\overline{PL}(m)$	$\overline{PB}(m)$	$\overline{PL}(m)$	$\overline{PB}(m)$	$\overline{PL}(m)$	$\overline{PB}(m)$	$\overline{PL}(m)$	$\overline{PB}(m)$	$\overline{PL}(m)$	$\overline{PB}(m)$	$\overline{PL}(m)$
1	.18	.18	.18	.14	.18	.11	.18	.07	.18	.04	.18	.0
2	.23	.23	.23	.18	.23	.14	.23	.09	.23	.05	.23	.0
3	.27	.27	.28	.22	.28	.17	.29	.11	.29	.06	.29	.0
4	.33	.33	.34	.27	.35	.21	.37	.15	.38	.08	.40	.0
5	.40	.40	.41	.33	.43	.26	.45	.18	.47	.09	.49	.0
6	.46	.46	.48	.39	.51	.30	.53	.21	.55	.11	.56	.0
7	.52	.52	.54	.43	.56	.34	.58	.23	.60	.12	.62	.0
8	.56	.56	.59	.47	.61	.37	.63	.25	.65	.13	.67	.0
9	.59	.59	.62	.50	.64	.39	.67	.27	.69	.14	.72	.0
10	.62	.62	.65	.52	.67	.40	.70	.28	.72	.15	.76	.0
11	.64	.64	.67	.54	.70	.42	.73	.29	.72	.15	.80	.0
12	.66	.66	.69	.55	.72	.43	.76	.30	.80	.16	.84	.0
13	.67	.67	.71	.51	.75	.45	.79	.31	.83	.17	.88	.0
14	.69	.69	.72	.58	.77	.46	.81	.33	.86	.17	.92	.0
15	.70	.70	.74	.59	.79	.47	.84	.34	.90	.18	.96	.0
16	.71	.71	.76	.61	.81	.49	.87	.36	.93	.30	1.00	1.00
17	.72	.72	.77	.62	.83	.50	.89	.39	.96	.50	1.00	1.00
18	.73	.73	.79	.63	.85	.52	.92	.46	.98	.70	1.00	1.00
19	.74	.74	.80	.64	.87	.54	.94	.54	.99	.84	1.00	1.00
20	.75	.75	.82	.65	.89	.56	.95	.64	1.00	.93	1.00	1.00
AVE	.56	.56	.59	.47	.62	.38	.66	.29	.69	.25	.72	.25

In section II we looked at two special cases dealing with the mean interarrival time between successive crisis calls. Figure 2 gives a numerical consideration of these cases. As a function of the mean interarrival time ($1/\alpha$), the sensitivities of the average blocking probability (PB) and the average loss probability (PL) are given for three ordinary loads. When $\alpha \to 0$ the mean interarrival time gets large and the values of $PB^{(m)}$ and $PL^{(m)}$ (and hence PB and PL) given by equation (30) serve as the lower bound on the average loss and blocking probabilities. Analogously, when $\alpha \to \infty$, equations (26) and (28) give upper bounds on these probabilities. Again, as we pointed out in section II, PB and PL are monotonically increasing in α.

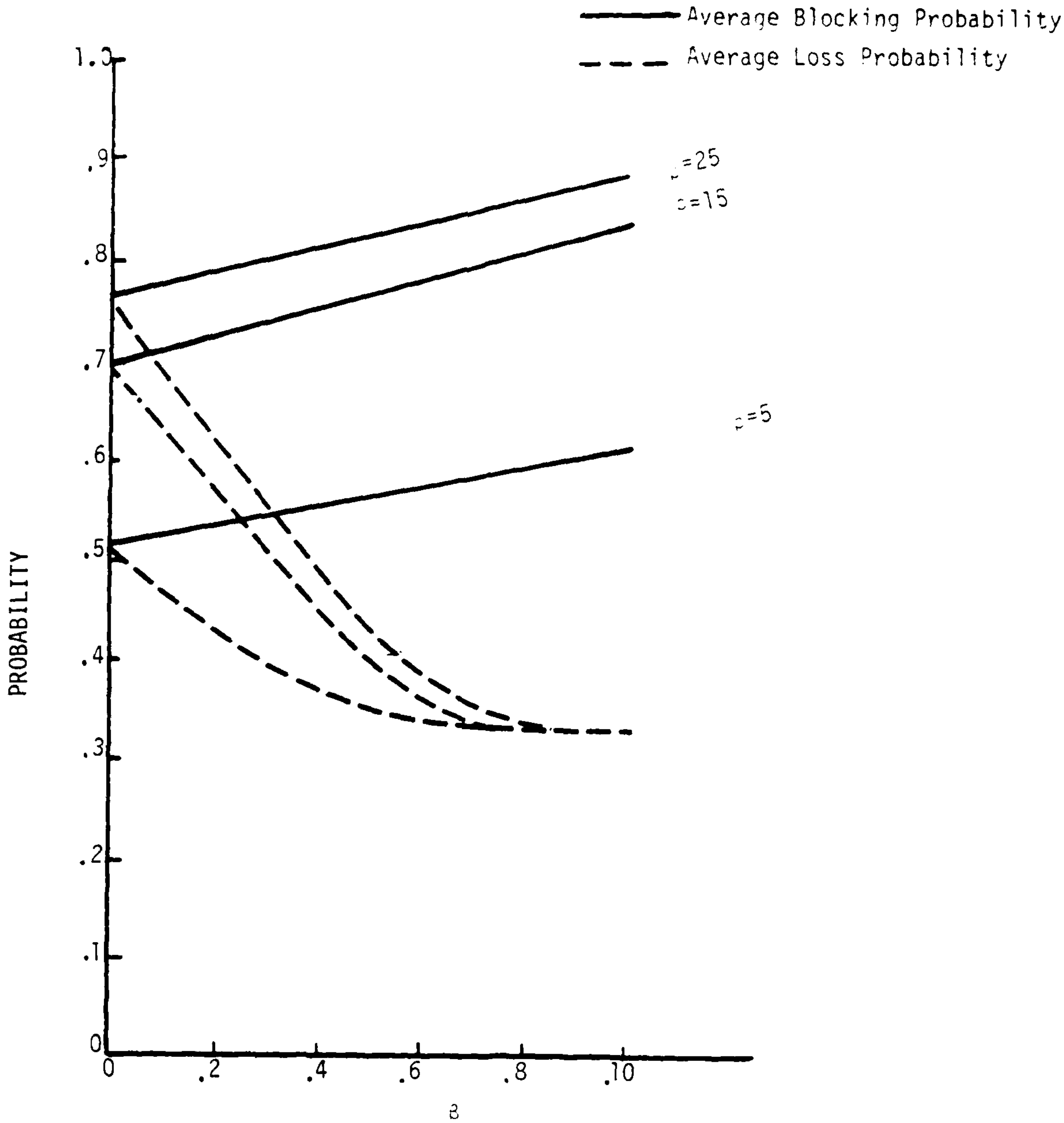

Figure 1. Sensitivities of Average Blocking and Loss Probabilities to Probability of Preemption and Ordinary Load (C=10, M=15, α=10)

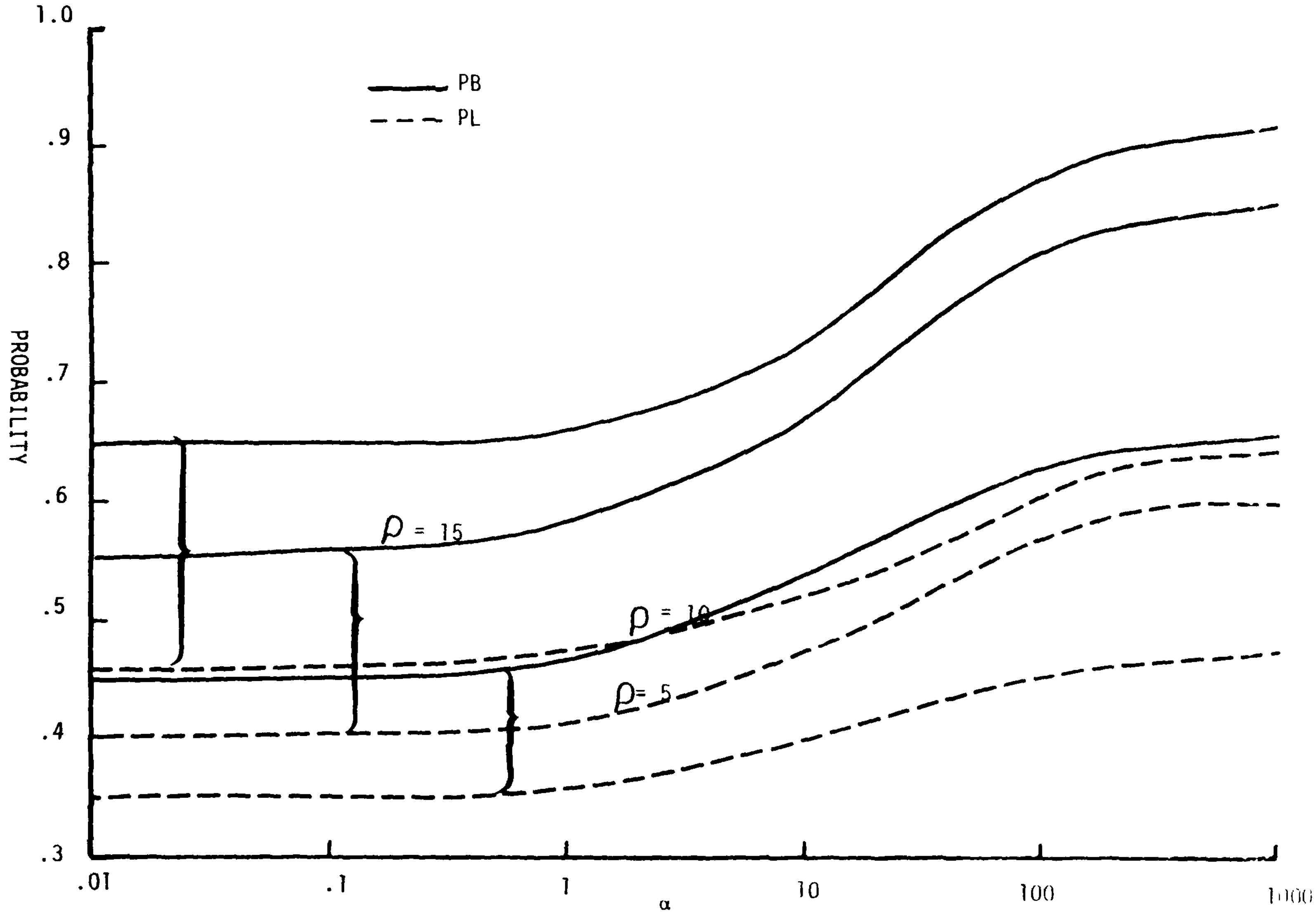

Figure 2. Sensitivities of Average Blocking and Loss Probabilities to the Mean Interarrival Time of Crises Calls (C=10, β=.3, M=15)

IV. CONCLUSIONS

In this technical note we present a mathematical model for the link behavior of crisis calls that may or may not preempt ordinary calls. This investigation was conducted in order to gain a better understanding of the behavior of the system with regard to the crisis calls. Armed with this information, we are now in a position to modify the DCEC network performance model so that computer-expensive event-by-event simulations will not have to be run for each Flash Non-Blocking Study.

Besides the development of the mathematical model, the two main findings of this report are the bounding of the average loss and blocking probabilities by expressions which are easily computed, and the monotonic property of these probabilities with respect to the mean time between crisis calls arrival.

We can elaborate a bit more on this second finding. For the case where $C=1$ and $\beta=0$, from equation (24) we have

$$PB=PL=\frac{1}{M}\left(M-1+\frac{\rho R_{11}[1]^{M-1}}{1+\rho}\right) \tag{32}$$

where

$$R_{11}[1]=\frac{\rho}{1+\rho}+\frac{\phi(\mu+\lambda)}{1+\rho}\ . \tag{33}$$

If the interarrival time is exponentially distributed then

$$\text{[illegible]} = \frac{\alpha}{\alpha+\mu+\lambda} = \frac{\frac{\alpha}{\mu}}{\frac{\alpha}{\mu}+1+c} \; . \qquad (34)$$

The monotonic behavior of PB and PL can be easily seen from equations (32), (33) and (34).

In fact, the factor α/μ is the driver in this monotonic behavior property. That quantity (α/μ) is the ratio of the mean ordinary call holding time to the mean interarrival time of the crisis calls. As the time period between crisis calls tends to become much longer than the average holding time of the ordinary calls, then $\alpha/\mu \rightarrow 0$ and PB and PL approach the lower bound. At the other extreme, as the time period between crisis calls becomes much shorter than the mean holding time of ordinary calls, congestion becomes more prevalent and PB and PL approach the upper bound.

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